Modeling Hot Carriers Quantum Well Solar Cells

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Abstract— The effects of carrier escape from quantum well (QW) and the interaction of hot electrons with crystal lattice are of importance to the physical understanding of OW hot carrier solar cells when the cooling dynamics in photo-excited structures affect the cell efficiency. The absorption of high-energy photons produces electron hole pairs with excess kinetic energy, which are dissipated to the lattice thru phonon scattering. These hot electrons alter the conversion efficiency in photovoltaic solar cells. We have studied hot electron effects in an $Al_xGa_{1-x}As$ / GaAs p-i-n structure with quantum wells are placed in the intrinsic region similar to the experimental device structure reported by others. Our results show that hot electrons lead to an increase in short circuit current. The increase in short circuit current is due to carriers escaping from the well without any significant recombination loss which leads to a higher cell efficiency. These results support the experimental data recently published by others.

Index Terms— Hot carriers, quantum wells, solar cells

I. INTRODUCTION

T is important to model devices with the precise I engineering of the band gaps to optimize the absorption incorporating quantum well layers into one or more of the subcells, allowing some leverage to design and fabricate devices for the absorption threshold to match the current generated in each subcell due to local absorption of the sun spectrum.

Single crystal quantum well solar cells are considered devices with high efficiency energy conversion used for application in both single and multijunction photovoltaic cells, as their capability outperform bulk devices under concentrated illumination close to the single gap

Shockley-Queisser limit for the same bulk materials[1].

In order to further increase the efficiency, the mechanisms behind the extraordinary performance need to be analyzed and understood. This paper present a model for the device behavior based on complex processes between localized and extended states, such as carrier capture and escape into and from quantum wells, which is described by the combination of macroscopic semiconductor transport equations and detailed balance rate equations.

The effects of carrier escape from QW and the interaction of hot electrons with crystal lattice are of importance to the physical understanding of QW hot carrier solar cells where the cooling dynamics in photo-excited structures affect the cell efficiency. The absorption of high-energy photons produces electron hole pairs with excess kinetic energy, which are dissipated to the lattice thru phonon scattering. Quantum wells were proposed as one approach in utilizing these carriers with excess kinetic energy. These hot electrons, so called hot carriers, alter the conversion efficiency in photovoltaic solar cells.

A microscopic numerical approach based on the non-equilibrium *Green's* function that takes into the consideration the interaction of quantum optics and dissipative quantum energy levels is used to model the quantum kinetic levels in photovoltaic's as it is discussed in [2-7]. Their modeling approach is capable of analyzing devices at reduced size and

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dimensionality with a high computation time limitations.



Figure 1. Band structure of a multi quantum well $Al_xGa_{1-x}As/GaAs$ cell. The wells are treated as single QW's with no overlap and coupling effect.

Our approach to device modeling for studying hot electrons in quantum well has been approximated by the numerical solution of the energy balance equations coupled with non-isothermal drift diffusion transport equations. The size quantization and the two-dimensional (2D) density of states are applied in the quantum wells. The assumptions made are: (a) quasi Fermi level is flat in the well, shown for the structure modeled in Figure 1, (b) the transition between 2D density of states and the bulk states (for calculating free carrier concentration) is abrupt, (c) in the QW, Boltzmann statistics describes the free and bound carrier concentrations, however the bound charge is dependent on the quantum well thickness, 2D density of states, and quantized energy level and wave-function, (d) a finite square well solution to wave function is used for the energy levels. The wells are treated as single *QW's* with no overlap and coupling effect. Various generation-recombination mechanisms are included, that is; electron-hole generation due to optical generation-recombination absorption, due to Shockley-Read-Hall (SRH), and band-to-band radiative mechanisms. Energy transfer among the charge carriers and crystal lattice is modeled by an energy relaxation lifetime.

II. DEVICE MODELING

The carrier rate equations, Poisson's equation, and auxiliary drift-diffusion current equations are solved coupled with the energy-balance equations to compute the internal electron temperature. The conventional drift-diffusion equations as well as Poisson equation are solved to describe the device current-voltage characteristics in the bulk region of the semiconductor, and outside the quantum wells. The device formulation follows. The Poisson equation:

$$\varepsilon_s \nabla^2 \cdot \psi \quad (x) - q [p(x) - n(x) + N_D(x) - N_A(x)] = 0$$
(1)

)

where $N_D(x)$ and $N_A(x)$ are position dependent donor and acceptor densities; n(x) and p(x) are the electron and hole densities determined from:

$$n(x) = N_c F_{1/2}(\eta_c)$$
(2)
and

$$p(x) = N_v F_{1/2}(\eta_v).$$
(3)

In the above equations, $F_{1/2}$ and F_0 are the Fermi Integral; η_c and η_v are the Plank potential for electron and hole.

The effective density of states in conduction band (N_c) and the effective density of states in valence band (N_v) for the bulk regions are:

$$N_{c,\nu}(x) = \frac{1}{4\pi^{3/2}} \left(\frac{2m_{n,p}^*(x)kT_{n,p}(x)}{\hbar^2} \right)^{3/2}$$
(4)

For the quantum well region the effective 2D density of states for conduction band (N_{cQW}) and valence band (N_{vOW}) are:

$$N_{c,vQW} = \frac{m_{n,p}^* k T_{n,p}}{\pi \hbar^2}$$
(5)

In the above equations, $m_{n,p}^*$ are density of states masses for electron and hole; k and \hbar are the Boltzmann and Plank constants, respectively; and T_n is the electron temperature and T_p is the hole carrier temperature.

In the QW the quantized energy levels for electrons (E_{OW_n}) and holes (E_{OW_n}) are computed from an analytical expression assuming a finite square well approximation:

$$\sqrt{\frac{2m_{QWn,p}^{*}E_{QWn,p}}{\hbar^{2}}} \tan\left(\sqrt{\frac{2m_{QWn,p}^{*}E_{QWn,p}}{\hbar^{2}}} \frac{L_{QW}}{2}\right) -\sqrt{\frac{2m_{BULKn,p}^{*}(V_{QWn,p} - E_{QWn,p})}{\hbar^{2}}} = 0$$
(6)

where $m^*_{OWn,p}$ are the electron and hole density of state masses in a quantum well and $m^*_{BULKn,p}$ are electron and hole density of state masses in the bulk material. In Eq. (6), V_{QWn} and V_{QWp} are conduction and valence heights in the QW, respectively, and L_{ow} is the well thickness. Both bound and free carriers are considered in the quantum well region.

non-isothermal drift-diffusion А current continuity equations for electron (J_n) and hole carriers (J_n) are considered:

$$\frac{1}{q} \nabla \cdot J_{n}(x) - (U_{SRH} + U_{b-b} - G_{OPT}) = 0$$
 (7)

$$\frac{1}{q}\nabla \cdot J_{p}(x) + (U_{SRH} + U_{b-b} - G_{OPT}) = 0$$
(8)
where

wnere $J_{-} = \mu_{-} n$

$$\left[(kT_n \nabla \eta_c + \nabla E_c) + (\frac{5}{2} + \nu_n) \frac{F_{3/2 + \nu_n}(\eta_c)}{F_{1/2 + \nu_n}(\eta_c)} \nabla kT_n \right]$$
(9)

and

$$J_{p} = -\mu_{p}p$$

$$\left[(kT_{p}\nabla\eta_{v} - \nabla E_{v}) + (\frac{5}{2} + v_{p}) \frac{F_{3/2 + v_{p}}(\eta_{v})}{F_{1/2 + v_{p}}(\eta_{v})} \nabla kT_{p} \right].$$
(10)

The constants v_n and v_p are the phonon scattering parameters [8]. In the above equations, the net generation/recombination terms are presented by band-to-band Auger recombination (U_{b-b}):

$$U_{b-b} = B(np - n_i^2)$$
(11)

and Shockley-Read-Hall recombination (U_{SRH}) :

$$U_{SRH} = \frac{np - n_i^2}{(p + n_i)\tau_n + (n + n_i)\tau_p}.$$
 (12)

The optical generation term (G_{OPT}) represents both the monochromatic single photon energy as well as solar spectrum [9].

For the simplicity of implementing the model, we have assumed that the transit time across the quantum well is zero so that the quantum well can be treated as a block in which the rate equations are not solved. Instead, the model uses the flux of carriers in and out of the quantum-well including generation-recombination terms in the well, where the bulk recombination terms are modified and extended to quantum well region:

$$U_{b-b} = \frac{1}{L_{QW}}$$

$$\left| \int_{0}^{L_{QW}} \psi_{n}(x) \psi_{p}(x) dx \right|^{2} B_{QW} (n_{2d} p_{2d} - n_{i,2d}^{2}).$$
(13)
and

$$U_{SRH} = \frac{1}{L_{QW}} \left| \int_{0}^{L_{QW}} \psi_{n}(x) \psi_{p}(x) dx \right|^{2}, \qquad (14)$$

$$\cdot \frac{(n_{2d} p_{2d} - n_{i,2d}^{2})}{(p_{2d} + n_{i,2d}) \tau_{n0} + (n_{2d} + n_{i,2d}) \tau_{p0}},$$

where n_{2d} and p_{2d} are total QW electron and hole concentrations, given by:

$$n_{2d} = N_{cQW} F_0 (\eta_c - \frac{E_{QWn}}{kT_n})$$
(15)
$$E_{QWn} = E_{CQWn} + E_{QWn}$$

$$p_{2d} = N_{vQW} F_0(\eta_v - \frac{E_{QWp}}{kT_p}).$$

(

In this model, the bound carriers in the well are calculated according to the probability distribution given by the wave functions, $\psi_{n,p}$. The computed separation of the confined and free carriers is used to determine the density of states for those carriers eligible for thermionic emission [10]. The thermionic emission model for heterostructure is applied to single quantum well device by including coefficient that considers such effects as the different effective masses, quantum mechanical

transmission into barrier, the quantum mechanical reflection at the interface.

In Eq.(12), n_i is the intrinsic carrier concentration in the bulk and in Eq. (13)-(14), $n_{i,2d}$ is the intrinsic carrier concentration in the quantum well.

Based on the Stratton's energy-balance derivations for hot carriers, the kinetic flux density equations for electrons (S_n) and holes (S_p) are:

$$\nabla \cdot S_{n,p}(x) + W_{n,p}(x) = 0$$
(17)
and for the lattice is given by

$$\nabla \cdot S_{L}(x) - W_{L}(x) = 0$$
(18)
where

$$S_{L}(x) = -\kappa(x)\nabla T_{L}$$
(19)

$$S_{n} = -\mu_{n}n\frac{kT_{n}}{q}$$

$$\left[(kT_{n}\nabla\eta_{c} + \nabla E_{c})(\frac{5}{2} + \nu_{n})\frac{F_{3/2+\nu_{n}}(\eta_{c})}{F_{1/2+\nu_{n}}(\eta_{c})} + \right] - \frac{E_{c}J_{n}}{q}$$
(20)
and

$$S_{p} = -\mu_{p}p\frac{kT_{p}}{q}$$

$$\left[(kT_{p}\nabla\eta_{v} - \nabla E_{v})(\frac{5}{2} + \nu_{p})\frac{F_{3/2+\nu_{p}}(\eta_{v})}{F_{1/2+\nu_{p}}(\eta_{v})} + \right] - \frac{E_{v}J_{p}}{q}.$$
(21)

In Eq. (17), W_n and W_p are the kinetic energy loss rate for electrons and holes (Joules per volume per second) which represent various kinetic energy exchanges among the carriers and lattice, and among the carriers themselves:

$$W_{n,p} = W_{n,p,SRH} + W_{n,p,b-b} + W_{n,p,RELAX}$$
$$-W_{n,p,OPT}$$
(22)

In Eq. (18), W_L is the total energy exchange with lattice:

$$W_{L} = W_{n,SRH} + W_{p,SRH} + W_{n,RELAX} + W_{p,RELAX}$$
(23)

In Eq. (22), the carrier energy loss rates due to SRH and band-to-band mechanisms for electrons (n) and holes (p) are given by:

$$W_{n,SRH} = \left(\frac{u_n}{n}\right) U_{SRH}$$
(24)
$$W_{p,SRH} = \left(\frac{u_p}{p}\right) U_{SRH}$$
(25)
$$W_{n,b-b} = \left(\frac{u_n}{n}\right) U_{b-b}$$
(26)
$$W_{p,b-b} = \left(\frac{u_p}{p}\right) U_{b-b}$$
(27)

where the kinetic energy for electrons (u_n) and holes (u_p) at a given temperature is $u_{n,p} = \frac{3}{2}k_B T_{n,p}$, related by carrier temperature and carrier populations, modeled by the interaction of carrier and phonon lattice.

In Eq. (23), the kinetic energy exchange among the lattice and carriers are modeled by energy relaxation lifetime for electrons $\tau_{n,E}$ and energy relaxation lifetime for holes $\tau_{p,E}$ [9]:

$$W_{n,RELAX} = \frac{u_n(T_n) - u_n(T_L)}{\tau_{n,E}}$$

(28) and

$$W_{p,RELAX} = \frac{u_p(T_p) - u_p(T_L)}{\tau_{p,E}},$$

(29)

where T_L is the lattice temperature and normally at room temperature.

The expression for the direct kinetic energy exchange among the electron and hole carriers based on the effective mass approximation due to optical generation for electron $(W_{n,OPT})$ and holes ($W_{p,OPT}$) in the bulk have the forms [10, 11]:

$$W_{n,OPT} = E_c G_{OPT} + \sum_{h \mathcal{G}_{OPT}} \frac{m_p^*}{m_n^* + m_p^*} (h \mathcal{G}_{OPT} - E_g)$$

$$G_{\mathcal{G},OPT}$$

(30)

and

$$W_{p,OPT} = -E_{v}G_{OPT} + \sum_{h\mathcal{G}_{OPT}} \frac{m_{n}^{*}}{m_{n}^{*} + m_{p}^{*}} (h\mathcal{G}_{OPT} - E_{g})$$

 $.G_{\mathcal{G},OPT}$

(31)

Equations (30) and (31) are modified for the QW and have the forms:

$$W_{n,OPT} = +(E_{n,QW} + E_c)G_{OPT}$$

$$+ \sum_{h\mathcal{G}_{OPT}} \frac{m_p^*}{m_n^* + m_p^*} (h\mathcal{G}_{OPT} - E_{g,QW})G_{\mathcal{G},OPT}$$
(32)
and
$$W_{p,OPT} = +(E_{p,QW} - E_v)G_{OPT}$$

$$+ \sum_{h\mathcal{G}_{OPT}} \frac{m_n^*}{m_n^* + m_p^*} (h\mathcal{G}_{OPT} - E_{g,QW})G_{\mathcal{G},OPT},$$
(33)

where $h\mathcal{G}_{OPT}$ is the energy of the absorbed incident photon.

It is noted that according to the Stratton's energybalance equations the flux density of electron kinetic energy and mean electron energy and electron temperature are strongly coupled [7]. The net kinetic energy loss rate for electrons (Joules per volume per second) represents various kinetic energy exchange among the carriers and lattice, and among the carriers themselves which is modeled by an average energy relaxation lifetime(τ_F). The release of energy by the carriers is modeled by the interaction of phonons and electrons and net energy loss rate due to Shockley-Read-Hall nonradiative mechanism and the loss of kinetic energy due to radiative recombination through emission of photons. These interactions causes the lattice to heat up. The resulting heat includes several sources such as Joules heat, generation-recombination heat, and Thomson and Peltier relationships [8, 9]. In the model implemented, we have assumed that hole carriers thermalize to the lattice temperature instantaneously. This assumption is valid as it is shown in the experimental work for determining the electron energy relaxation lifetime in Al_xGa_1 . "As/GaAs QW p-i-n devices.



Figure 2. Band structure of a single quantum well $Al_xGa_{1-x}As$ /GaAs cell. The multi quantum wells are treated as single QW's with no overlap and coupling effect.

III. RESULTS AND DISCUSSIONS

A numerical simulation approach is employed to study the effects of carrier escape from quantum well and the interaction of hot electrons with crystal lattice in QW hot carrier solar cells where the cooling dynamics in photo-excited structures affect the cell efficiency. Simulation data show that hot carriers escape from multi quantum well results in a higher maximum conversion point in structures with long energy relaxation lifetime which may lead to a higher efficiency in hot electron solar cells. The band structure of an AlGaAs/GaAs single cell is shown in Figure 2. The QW is embedded in the center of the p-i-n diode. In detailed energy balance equations the non-isothermal current density is solved to determine the electron temperature. The electron temperature distribution for different energy relaxation lifetime is shown in Figure 3, where the energy transfer among the charge carriers and crystal lattice is modeled by an average energy relaxation lifetime. The temperature in the well is constant due to quantized energy level of the well. The average electrons populations stay hot for a longer energy lifetime due to less of energy loss to the lattice. Figure 3 shows that in a multiquantum well structure hot electrons with longer energy relaxation time tend to stay at higher energy before relaxing to the lattice.



Figure 3. In a multiquantum well structure hot electrons with longer energy relaxation time tend to stay at higher energy before relaxing to the lattice.



Figure 4. The Energy relaxation loss rate for

different relaxation energy lifetime.

The net energy loss rate for a single QW p-i-n diode is shown in Figure 4. In addition to this rate, the limiting *Shockley-Read-Hall* recombination rate per unit volume is also considered. The result of simulation for the single QW p-i-n diode showing *Shockley-Read-Hall* recombination energy rate at different energy lifetime is shown in Figure 5. This loss rate is higher for samples with longer energy lifetime since this parameter controls the average population of the electrons in the QW. The result of simulation for electron distribution at different energy relaxation lifetime is shown in Figure 6. A higher carrier density in the well translates to a higher *SRH* energy loss rate.



Figure 5. SRH energy loss rate in the QW for different carrier lifetimes. Both free and bulk SRH rates are considred in the model.



Figure 6. The simulated result for electron concentration in QW by increasing energy relaxation lifetime from 1ps to 1ns. The figure on the right shows the zoomed section in QW region of the p-i-n diode.

We have studied an $Al_xGa_{1-x}As / GaAs$ structure with several quantum wells placed in the intrinsic region similar to the device described in reference [5]. The quantum wells are modeled with 60 Å $Al_xGa_{1-x}As$ barriers and 85 Å GaAs wells and the Al composition is x=0.35. The p-type and n-type layers are also made out of the barrier $Al_xGa_{1-x}As$ material. Parameters used in the simulation are taken from published literature summarized in Table I. The numerical model presented in this work is aimed at improving the existing device models for the characterization of nano-structure hot carrier solar cells.

TABLE I DEVICE PARAMETERS USED IN 30 QWAl_xGa_{1-x}As /GaAs HOT FLECTRON SOLAR CELL

HOT LEECTRON BOLAR CELL	
Parameters V	Value
p^+ GaAs cap thickness0 p^+ GaAs cap doping1	0.02 m $1.8 \times 10^{18} \text{ cm}^{-3}$

p layer $Al_xGa_{1-x}As$ thickness	0.3 m
p layer $Al_xGa_{1-x}As$ doping	$7 \text{ x } 10^{17} \text{ cm}^{-3}$
<i>i</i> - region thickness	0.48 m
<i>n</i> layer $Al_xGa_{1-x}As$ thickness	0.3 m
<i>n</i> layer $Al_xGa_{1-x}As$ doping	$7 \text{ x } 10^{17} \text{ cm}^{-3}$
Quantum well's (GaAs) thickness	85 Å
Quantum barrier's $(Al_xGa_{1-} As)$ thickness	60 Å
Al composition material, x	0.2-0.46
SRH lifetime	1-30 ns
Surface recombination velocity (S_{n}, S_{p})	1 x 10 ⁵ cm/s
Energy relaxation lifetime (τ_E)	1-10 ps

Multi quantum well structures can slow down the cooling process (carriers with long energy relaxation lifetime) by reducing the phonon scattering effects. In this process, the rate at which the carriers are lost to the lattice is slowed down so called carriers escape. As a result, quantum well theoretically can enhance solar cell efficiency in devices by reducing the non-radiative recombination effect.

Figure 7(a) shows the simulated current-voltage characteristics of QW photodiode with 30 periods for three different values of energy relaxation lifetime, computed in an air mass 1.5 spectrum. These results show that hot electron effects lead to an increase in short circuit current. The increase in short circuit current is due to hot carriers escaping from the wells without any significant *SRH* recombination. As shown in Figure 7(b), the maximum power point is higher in a device with longer energy lifetime. Long energy relaxation lifetime slows the cooling dynamics and increases the rate of carriers escaping from the quantum

wells, which may lead to a higher cell efficiency. These results support the experimental data recently published by others [2, 5].



(a) Current-voltage characteristics



(b) Conversion power output

Figure 7. A 30 QW Al_{0.35}Ga_{0.65}As/GaAs p-i-n diode simulated in an air mass 1.5 spectrum with an energy relaxation lifetime of $\tau_E = 1$, 5, and 10 ps, (a)

Current-voltage characteristics, (b) Conversion power output.

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