Robot Collision Avoidance with a Guaranteed Safety Zone and Randomized Symmetry Breaking

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***Abstract*—Collision avoidance of moving systems is a well-studied problem. The use of an Artificial Potential Field function is a popular approach to compute in real time a path that avoids collision between agents. It involves the minimization of a weighted sum of an attractive force and a repulsive force. Previous studies consider these weights to be fixed design parameters, to be determined experimentally. In particular, these parameters do not change during the run of the algorithm. Our main result is based on the observation that by dynamically changing these parameters one can obtain a guarantee on a minimum safety distance between the agents. Specifically, if the agents compute their path by minimizing the potential field with properly chosen weights, there will always be a guaranteed safety distance between each pair of agents. Our earlier studies show promising experimental results and we extended the studies on avoiding trajectory symmetry.Our simulation validates our model and demonstrated its effectiveness for a group of non-cooperative agents moving in a small area.**

***Index Terms*—Collision Avoidance, Guaranteed Safety, Unsymmetrical, Unconstrained Optimization**

I. INTRODUCTION

Collision avoidance is a critical issue in any moving system. The cost of collision ranges from damages to equipment to the loss of human lives. Traditionally, vehicles and airplanes are controlled solely by a human operator. The possibility of collision is largely dependent on the operator's ability to respond to a situation effectively. Today, aside from high volume traffic, the addition of autonomous and semi-autonomous systems has made the problem even more complex. Meanwhile, many human operators, particularly those on the road, are more distracted than ever due to talking and texting on their cell phones. The need for effective collision avoidance control is tremendous. For simplicity, we will focus our discussion on vehicles, though many of the issues and models are applicable to other systems.

Many researchers have studied and proposed various models of collision avoidance. See, e.g., [1] and reference therein. The popular artificial potential field approach [2] is one such example. The environment is created as a vector field of attractive and repulsive forces. The vehicle's goal is an attractive force to it while obstacles or other vehicles are repulsive forces. These forces increase when a vehicle is close to its goal or an obstacle (or a neighbor vehicle) and decreases when the vehicle and its goal or obstacles are further away. The classical potential field method suffers from local minima issue. Many works, such asthose using Harmonic potential function [3],have resolved thislimitation. Furthermore, some of them are even able to guarantee collisionavoidance [4],[5],[6] under specific scenarios.One of the issues these workshave to address is how to react in time under high velocity and within theconstraints imposed by a vehicle's dynamics [7]. The issue is furthercomplicated if there is a crowding of the vehicles. Many times, admissiblepaths are not obtainable at the last moment [8].

Another approach includes a decision making component in their model. Forexample, trajectory planning, generally receives input parameters such asvehicle dynamics, location of obstacles and goal, and computes either a fullor partial motion plan for a vehicle system [9]. There are two issues with planners - first, it incurs high computational costs. Second, it is sensitiveto unexpected changes in the environment. Another example is the coordinationof multiple vehicle systems.See, e.g., [10],[11] and reference therein. Coordination is performed either by a centralized planner thatfactors all the vehicles' information and sends each vehicle a motion plan, or if the centralized planner is not available, the coordination is performeddirectly between these vehicles. One of the issue with planners is maintaininga stable communication link and high computational. Another deliberateapproach is predictive models. Predictive model performs a “look ahead”, verymuch like a human operator would do. A common predictive control model thatone can find in literature is the Receding Horizon (HC) controls [12],[13],also known as Model Predictive Control(MPC) [14]. RH is used as an optimizersince the key strength of this model is that it only looks a few time stepsahead for each move the vehicle makes instead of planning the wholeroute. Nevertheless, this method cannot guarantee collision avoidance as thecollision point may be right after its look-ahead time [15]. Thus the method

This paper is an extension from the ATAI 2012 conference paper.

Manuscript received February 28, 2013. This work was supported in part by the NSF-GK 12 program.

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DOI: 10.5176/2251-3043_3.1.225

issensitive to initial conditions of the vehicles. There are a lot more othercollision avoidance techniques that are not described here, such as road-map [16] and distance transform [17].

In the previous decades, many motion models did not consider dynamics in their design. These models raised issues when they are implemented into physical systems. For example, these models may expect a vehicle to arrive at a specific location in a specific time frame to avoid collision but the vehicle control system has dynamic constraints that prevent it from executing these instructions in time. Hence, many researchers started to shift their focus to incorporating physical laws into their motion models and produced a lot of excellent work. However, these models may not be applicable if a system is not constraint by dynamics.Our work is focused on how to produce a motion control model at a higher, more abstract level, such that systems governed by dynamics and systems not governed by dynamics, can both utilize.

In review of many works, we argue that by making sure our model guarantees a user defined minimum safety distance; we have indirectly incorporated dynamic and kinematic constraints. For example, many airplane safety regulations require some specific minimum safety distance to avoid colliding with another plane or obstacle. The distance is computed based on an airplane's kinematics, dynamics, turbulence, and various other factors. Similar techniques can be applied to compute safety distances for vehicles and vessels by their motion controllers or some lower level modules. For example, [4],[6] defined collision and avoidance regions for an agent based on control laws. Our adaptive collision avoidance motion model takes in the distance as a parameter and guarantees the vehicle will always maintain the specified safety distance at any point.

Our method utilizes the weighted artificial potential field approach with unconstrained optimization. We chose the potential field approach due to its simplicity. The approach consists of two forces - the attractive force to the goal and the repulsion of its neighbors. Generally, these forces are represented as objective and avoidance functions. In most of the work we reviewed, there is also a weighted term on the functions. While there are many techniques that use weights in their models, most of these models emphasize on custom objective and avoidance functions while the weights play a secondary role. Hence, these weights are either manually configured or a product of inverse square distance between a vehicle and its goal or obstacle. See e.g.[18],[4],[19],[6] and reference therein. We argue that the choice and computation of these weights can greatly affect the outcome and results of the collision avoidance. Hence our technique takes the opposite approach - we use basic objective and avoidance functions and attach custom derived weighting parameters to drive these functions. The focus of this paper is to present a model that derives these weights.

Some of the researchers such as Park et al [20] and Barnes et al [21] have proposed dynamic weights. Park et al's model derived the weight through "reverse engineering". The model requires the system to imitate the human motion from start to goal. Then the system backtracks to compute the weight at each location from goal to start. This model cannot be used in an autonomous system or systems with no human intervention. Barnes et al's model is closest to ours. Their weights are derived from "limiting functions" that decreases to zero as the vehicle gets close to the goal or avoids obstacle. However, while their model has a minimum distance parameter, the distance is not guaranteed as there are situation where the avoidance vector magnitude is zero.

We tested our model on a group of non-cooperative agents moving in a small area. The test results validate our model and demonstrate its effectiveness. For simplicity of discussion, we will use the term "agents" hereafter to refer to any vehicles, robots or autonomous moving objects.

II. MODEL OVERVIEW

A. Background

We consider n agents $a_1 \ldots a_n$moving around and attempting to reach their respective goals.We use the following notation:

x: the location of the agent
g : the goal location of the agent}
S: the (squared) safety distance that the agent wants to keep
x_j: the location of Agent j that should be avoided, $j=1..m$
g_j: the goal of Agent$j$

III. MODEL SPECIFICATION

At any given time we denote the location of the ith agent by x^t, and the location of its target by g^t. The squared distance between x^t and g^t is given by:

$$L_i^t = \| x^t - g^t \|^2 \tag{1}$$

We will use (1) as our attractive potential force in our computation later on.In moving toward its goal, agent i attempts to avoid collisions with other agents by keeping a minimum safety distance S between itself and all other agents.

This can be expressed by the following constraint:

$$\min_j \| x^t - x_j^t \| \geq S \tag{2}$$

Our main result is a simple procedure that minimizes (1) subject to (2) utilizing a gradient based optimization [22] procedure.In order to apply gradient based optimization we need to replace (2) by a smooth function.We propose the following:

$$L_a^t = \sum_{j=1}^{m} \frac{1}{\| x^t - x_j^t \|^2} \tag{3}$$

We will use (3) as our repulsive potential force.

Observe that a small value of L_i^t indicates that $a_i$is near its goal, and similarly, a small value of L_a^tindicates that $a_i$is far from the other agents. This suggests that we attempt to minimize their weighted sum. Define L^t to be their weighted sum:

$$L^t = L_i^t + \delta^t L_a^t \tag{4}$$

As we show, it is possible to iteratively update the value of δ^tso that minimizing L^t produces a minimization of L_i^t that satisfies the constraint (2).

In order to minimize L, we first compute the derivatives of the attractive and repulsive potentials, which are also known in many literatures as the objective and avoidance functions.

The derivative of the function (1) and (3):

$$\nabla L_i^t = 2(x^t - g^t)$$

$$\nabla L_a^t = -2\sum_{j=1}^{n} \frac{x^t - x_j^t}{|x^t - x_j^t|^4}$$

The derivative of the potential field function is:

$$\frac{1}{2}\nabla L^t = x^t - g^t - \delta^t \sum_{j=1}^{n} \frac{x_j^t - x^t}{|x_j^t - x^t|^4}$$

Direction the agent should go (negative gradient):

$$Direction = g^t - x^t + \delta^t \sum_{j=1}^{n} \frac{x^t - x_j^t}{|x_j^t - x_i^t|^4}$$

Recall from (3), the repulsive potentialat time t is:

$$L_a^t = \sum_{j=1}^{m} \frac{1}{\| x^t - x_j^t \|^2}$$

For simplicity, we use d to represent the difference term. So the repulsive force can be written as:

$$L_a^t = \sum_j \frac{1}{d_j^t}, \quad d_j^t = |x^t - x_j^t|^2 \tag{5}$$

The key idea that we introduce is the fact that the guarantee of a safety distance S can be made by properly adjusting the weight constant δ^t. We begin by introducing several simple lemmas:

a) Lemma 1: If $L_a^t \le \frac{1}{S}$ then $d_j^t \ge S$ for all j.

b) Proof::

$$\frac{1}{d_j^t} \le \sum_j \frac{1}{d_j^t} \le \frac{1}{S}$$

Since the potential to be minimized is L^t and not L_a^t,we wish to determine what value of L^twould providea guarantee of a safety zone.Suppose we know that B^t, is lower bound on the value of L_i^t. Using such bound we can generalize the result of Lemma 1 as follows:

c)Lemma 2: If $B^t \le L_i^t$ *and* $L^t \le B^t + \frac{\delta^t}{S}$*, then* $d_j^t \ge S$ *for all j.*

d) Proof::

$$L_a^t = \frac{L^t - L_i^t}{\delta^t} \le \frac{B^t - L_i^t + \delta^t / S}{\delta^t} \le \frac{\delta^t / S}{\delta^t} = \frac{1}{S}$$

The result of the lemma now follows by applying Lemma 1. Observe that the value of δ^t is a free parameter. From Lemma 2 it follows that the following constraint is necessary and sufficient to satisfy the second condition of the lemma:

$$\delta^t \ge (L^t - B^t)S \tag{6}$$

Now suppose that at time t_1 the agent has a current value of δ^{t1}. Since the agent can observe its surrounding, it can calculate the values of $L_i^{t1}$and L_a^{t1}. The value of L^{t1} is determined from Equation(4). We wish to determine a good value of $\delta^{t2}$for $t_2 > t_1$, so that the minimization of $L^{t2}(x)$= $L_i^{t2}(x) + \delta^{t2} L_a^{t2}(x)$ would guarantee the existence of the safety zone S around the agent. First assume that the minimization reduces the value of the potential.

e)Lemma 3: If $B^{t_2} \le L_i^{t_2}, L^{t_2} \le L^{t_1}$, and

$$\delta^{t_2} \ge (L^{t_1} - B^{t_2})S \tag{7}$$

then $d_j^t \ge S$ for all j.

f) Proof::

$$\delta^{t_2} \ge (L^{t_1} - B^{t_2})S \ge (L^{t_2} - B^{t_2})S$$

and the proof follows by applying Lemma 2.

Since we are only interested in satisfying the safety zone constraintwe should select the smallest δ that comes with the safety zone guarantee.

This is the value obtained with an equality in (7):

$$\delta^{t_2} = (L^{t_1} - B^{t_2})S \tag{8}$$

The condition $L^{t_2} \le L^{t_1}$, one of the conditions in Lemma 3, is sufficient but not necessary.Even if it does not hold the minimization outlined in the lemma may produce a solution satisfying $L_a^{t2} \le 1/S$ which would guarantee the safety zone around the agent. If this condition does not hold, the value of δ^{t2} should be increased. The following argument suggests a heuristic for increasing the value. Consider the following choice for δ^{t2}:

$$\delta^{t_2} = (L^{t_1} - B^{t_2} + K)S \tag{9}$$

Suppose the minimization of $L^{t2}(x) = L_i^{t2}(x) + \delta^{t2} L_a^{t2}(x)$ gives the minimum value as x_1, but $L_a^{t2}(x_1) > 1/S$.Then:

$$L^{t_2}(x_1) = L_i^{t_2}(x_1) + \delta^{t_2} L_a^{t_2}(x_1) > L_i^{t_2}(x_1) + \frac{\delta^{t_2}}{S}$$

$$= L_i^{t_2}(x_1) + L_i^{t_1} - B^{t_2} + K \geq L_i^{t_1} + K$$

This cannot hold at the point x_1 if $K = L^{t2}(x_1)$.Setting $\delta^{t2} = (L^{t1} - B^{t2} + L^{t2}(x1))S$ and minimizing of $L^{t2}(x)$ would produce a minimum point x_2. Here, either $L_a^{t2}(x_1) \leq 1/S$, which guarantees the safety zone,or we must have:

$$L^{t_2}(x_2) > L_i^{t_1} + L^{t_2}(x_1)$$

Continuing with this procedure would increase the value of δ.If a solution is not found for a large δ, it means thatthere is no solution satisfying $L_a^{t2}(x_1) \leq 1/S$,even if L_i^{t2} is ignored. The procedure described above suggests the following algorithm that is executed by an agent to determine where to move.

Input: The value of *x, g, S* for the agent.
The previously used value of δ^{t1}.
The values of x_i and g_i for all neighbors that are closeenoughto be considered.
A maximum number of iterations *N*.

Output: The new location.

1. Compute the values of $L_i^{t_1}$, $L_a^{t_1}$, L^{t_1}, B^{t_2}.
2. Set $\delta = (L^{t_1} - B^{t_2})S$, and $\delta^{t_2} = \delta$.
3. Iterate at most *N* times:

3.1 Apply a gradient optimization method to compute x^* that minimizes $L^{t_2}(x) = L_i^{t_2}(x) + \delta^{t_2} L_a^{t_2}(x)$.

3.2 If $L_a^{t_2}(x^*) < 1/S$, add a very small random value γ to x^*, $x^{*\prime} = x^* + \gamma$ such that $L_a^{t_2}(x^{*\prime}) < 1/S$ still holds. Return $x^{*\prime}$ and terminate the algorithm.

3.3 Otherwise update $\delta^{t_2} = \delta + L^{t_2}(x^*)$ and continue with the iterations.

Cases in which the algorithm will not find a suitable x^* will be discussed later.

The implementation of the algorithm requires a formulafor the bound B^{t2}.This plays a similar role to that of admissible heuristics in AI.Among all possible bounds we would like to select it as largeas possible.The following lemma gives the result that we have used in our experiments.

g) Lemma 4: Let σ be the maximum speed of the agent.Then

$$B^{t_2} = \max\{(\sqrt{L_i^{t_1}} - (t_2 - t_1)\sigma)^2 \;, 0\} \tag{10}$$

satisfies:

$$B^{t_2} \leq L_i^{t_2}(x)$$

h) Proof:: A lower bound on the value of$L_i^{t2}(x)$ is obtained if the motion between t_1 and t_2 is directly toward the goal g^{t2}, ignoring all obstacles.Let r be a unit vector specifying the direction.This observation implies:

$$|g^{t_1} - x^{t_1} - (t_2 - t_1)\sigma r|^2 \leq L_i^{t_2}$$

Applying the triangle inequality to the left hand side we get:

$$\sqrt{L_i^{t_1}} - (t_2 - t_1)\sigma$$
$$= |g^{t_1} - x^{t_1}| - (t_2 - t_1)\sigma |r| \leq |g^{t_1} - x^{t_1} - (t_2 - t_1)\sigma| r|$$

So that

$$\sqrt{L_i^{t_1}} - (t_2 - t_1)\sigma \leq \sqrt{L_i^{t_2}}$$

To prove the lemma it remains to observe that trivially $0 \leq L_i^{t_2}(x)$.

A. Limitation of the algorithm

There are three cases in which the proposed algorithm fails to determinea move which guarantees a safety zone even after running a large numberof iterations.

Case 1: There is no solution. Collision is unavoidable.

Case 2: Collision is avoidable, but the minimizationalgorithm fails because it gets stuck in a localminimum.

Case 3: There is no solution satisfying the constraint$L_a \leq 1/S$, butthere is a solution satisfying$d_j \geq S$ for all *j*.

While there is nothing that can be done in Case 1, there are standard approaches to resolve Case 2. Specifically, one can employ other minimization algorithms(e.g., simulated annealing), or introduce a small amount of randomness in the solution.

Case 3 indicates situations where our approach fails whileother algorithms based on the idea of artificial potentialmay work. We would like to point out that such cases are rare. They only occur under the following condition:

$$\sum_j \frac{1}{d_j} \gg \frac{1}{min_j d_j}$$

But this is unlikely to hold near collision, since then the right hand side is very large by itself.

IV. SIMULATION

A. Setup

We simulated the motion of various agents using our proposed model. Each agent starts at some initial location represented in (x,y) coordinates. The objective of each agent is to travel to the goal coordinates. There are three input parameters to the system - the minimum safety distance S, the speed of the agents, and the maximum iterations.

B. Settings

In our previous experiments [23], we were able to demonstrate that the agents will not violate the specified minimum safety distance using our model (Figure 1 and 2).

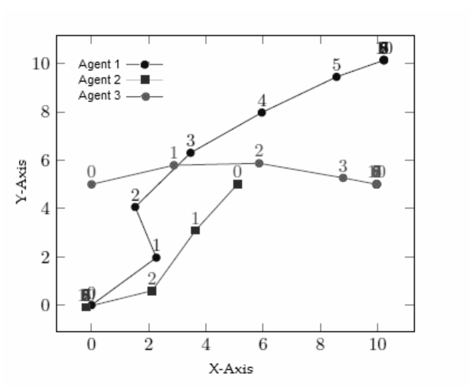

Figure 1 Three agents moving

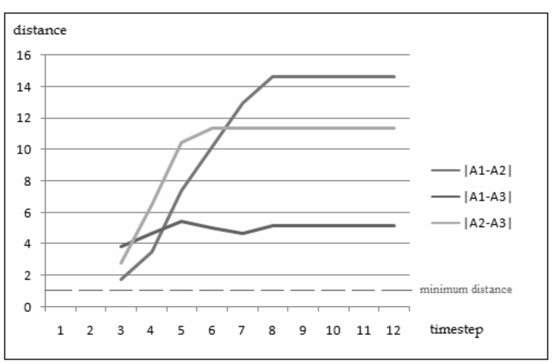

Figure 2 Minimum distance between agents

We extend the experiments of our previous algorithm to observe the effects of specific trajectories and goal location. We conducted two types of experiments - parallel trajectories and goal convergence. For the parallel trajectories test, the agents travel in direct, opposite direction (head-on collision) at the same speed and distance. For the convergence test, the agents have different trajectories but heading for the same goal at the same time.

Experiment *4: Parallel Trajectories, 2 agents head-on

Agent 1: start=(0,5), goal=(10,5), speed=1, S=1
Agent 2: start=(10,5), goal=(0,5), speed=1, S=1

Experiment 5: Parallel Trajectories, 4 agents head-on

Agent 1: start=(0,2), goal=(10,2), speed=1, S=1
Agent 2: start=(10,2), goal=(0,2), speed=1, S=1
Agent 3: start=(0,5), goal=(10,5), speed=1, S=1
Agent 4: start=(10,5), goal=(0,5), speed=1, S=1

Experiment 6: Same Goal, 2 agents

Agent 1: start=(0,0), goal=(10,0), speed=1, S=1
Agent 2: start=(10,10), goal=(10,0), speed=1, S=1

*Experiment 1,2, and 3 are shown in the ATAI 2012 conference paper.

C. Analysis

In our previous algorithm, there was an inherent symmetry issue. This issue occurs when two agents are coming at exact opposite position, heading directly at each other and their goal is each other's' starting point (Experiment 4). As shown in Figure 3a, a stalemate occurred. However, this does not occur when there are two pairs or more of such agents (Experiment 5, Figure 3b) as each agent are now influenced by three agents, two with different positions. To address the symmetry issue, we added a small random value to each location calculation. Step 3.2 in the new algorithm (Section III) reflects this modification from our previous paper.

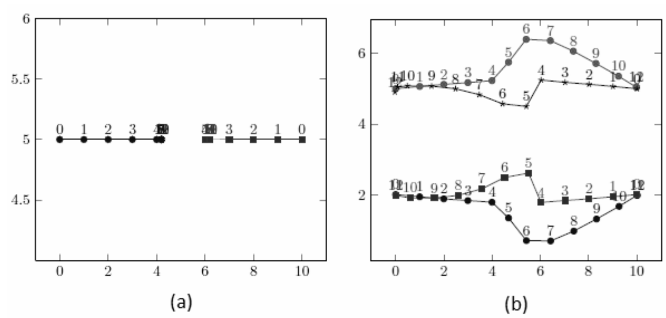

Figure 3 Symmetry observed in two agents head-on (left)

The added randomness breaks the symmetry and the two agents altered their trajectories and avoided each other (Figure 4a) without violating the minimum safety distance (Figure 5). We repeated the previous experiments with the modified algorithm and the results show that the added random value barely altered the trajectories from the previous experiments (Figure 4b).

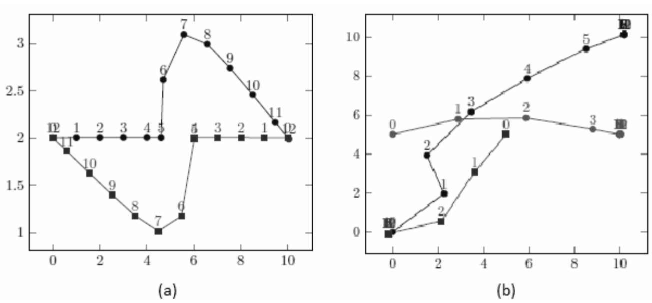

Figure 4 Symmetry avoided and trajectory preserved

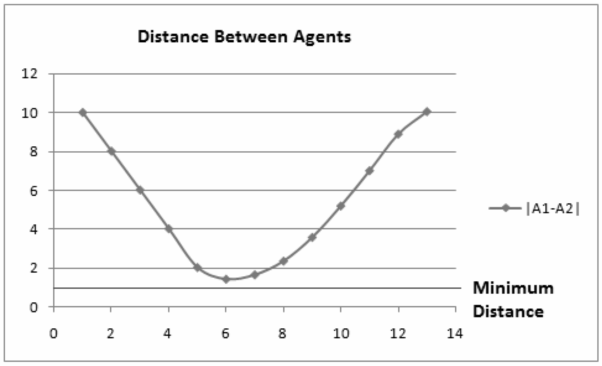

Figure 5 Safety distance maintained with randomness

However, one limitation still exists in the new algorithm. When the agents have the same goal and are all arriving at the same time, the agents will stalled at the perimeter of the goal as they cannot converge to the same position (Figure 6).

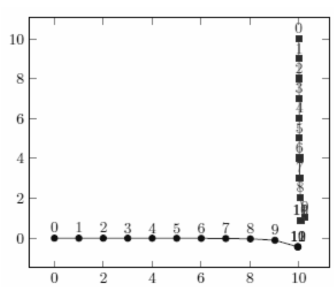

Figure 6 Agents with the same goal

V. Conclusion

We presented a model that adaptively avoids collision amongst multiple agents moving in the same space using a novel weighted potential field function with unconstrained optimization technique. The model guarantees a minimum safety distance between these agents while minimizing the diversion from their goal trajectories. The algorithm also prevents agents from entering a stalemate due to symmetrical trajectories. Furthermore, because our technique is based on artificial potential field function, it does not require coordination or communication between the agents. Apart from reducing communication overhead, it can also be utilized in situation where there are communication interference or non-cooperating agents. There are still much work to be done, future plan includes extending this model with prediction strategies, investigating the benefits of incorporating game theory, particularly in environments with antagonist and non-antagonist agents.

Acknowledgment

The authors would like to thank Erick Rodriguez-Seda and Mark W. Spong for fruitful discussions andsuggestions.

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