Energy-Saving Periodic Flight at Transonic Speeds

G. Amouyal and D. Weihs

***Abstract*: We examine possible energy saving by periodic changes in speed and altitude relative to steady state cruise, in the transonic regime. We develop a theoretical model of two-phase periodic flight with altitude variation in the transonic regime and compare it with a steady horizontal flight at the same average speed. The model predictions are verified by wind tunnel experiments. For example, by periodically accelerating from M=0.7 to M=1.1 while losing altitude, and then climbing back to the original height while decelerating, one can achieve savings of about 20% relative to steady flight at the same average speed and constant altitude.**

Keywords: Flight Mechanics, Transonic, Periodic Motion

I. INTRODUCTION

Energy efficient flight is of great economic and environmental importance. The trade-off between the constraint of the high transonic drag, and the requirement of high speed has resulted in high subsonic cruise speeds for transportation, with the "sonic barrier" – (the large increase in drag at transonic speeds [1] limiting these speeds. In the present paper we show a method of increasing average cruise speed in this regime, with lesser drag.

The typical assumption is that the most economical way to cruise is in in steady, rectilinear flight, with very gradual altitude and speed changes to accommodate weight loss due to fuel usage.

Previous studies in various other speed ranges of aircraft (Speyer, 1976, Grimm & Weil, 1986, Sachs & Christodoulou 1986, Menon & Sweriduk, 2007) have indicated a possibility of energy sparing by applying cyclic unsteady motion. Periodic motion has been shown also to save energy in animal locomotion (Weihs, 1974, Videler & Weihs 1982). Therefore it is interesting to study such periodic motion for transonic flight and see if there is a possibility of energy saving at transonic speed also.

We developed a relatively simple two-stage model, of acceleration from subsonic to supersonic speeds while losing altitude and then climbing back up again, to original speed and height. In order to check the surprisingly good results we obtained, we compared the results of this model to wind tunnel experiments on a standard body-wing model- the AGARD –B calibration model used in most wind tunnels worldwide, for which highly accurate experimental data exist.

The equations of motion for the two-stage maneuver are developed here, for comparison with steady flight at the same average speed and constant altitude. Assuming no lateral motions, we can limit ourselves to a two dimensional analysis.

II. FIRST STAGE: DESCENT

The equations of motion, in inertial coordinates are

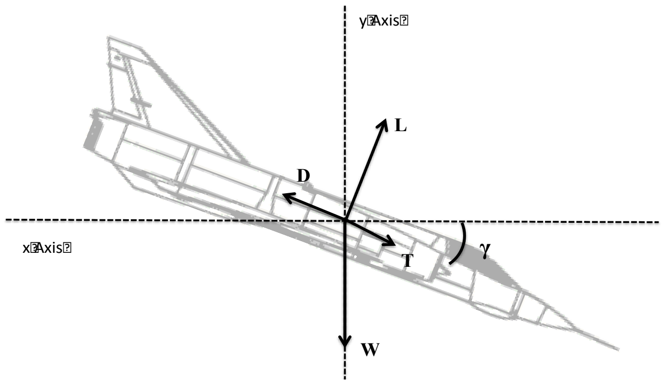

Figure 1. Forces during descent

$$\sum F_x = T\cos\gamma - D\cos\gamma + L\sin\gamma = ma_x \tag{1}$$

$$\sum F_y = L\cos\gamma - W - T\sin\gamma + D\sin\gamma = ma_y \tag{2}$$

$$a_x = \frac{\partial V_x}{\partial t} = \frac{T\cos\gamma - D\cos\gamma + L\sin\gamma}{m} \tag{3}$$

$$a_y = \frac{\partial V_y}{\partial t} = \frac{L\cos\gamma - W - T\sin\gamma + D\sin\gamma}{m} \tag{4}$$

$$\frac{\partial V}{\partial t} = \sqrt{a_x^2 + a_y^2} \tag{5}$$

$$V_y = \frac{\partial h}{\partial t} = V\sin\gamma \tag{6}$$

$$V_x = \frac{\partial x}{\partial t} = V\cos\gamma \tag{7}$$

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$$\frac{\partial V}{\partial t}\frac{\partial t}{\partial x}=\frac{\partial V}{\partial x}=\frac{\sqrt{a_x^2+a_y^2}}{V\cos\gamma} \quad (8)$$

$$\sin\frac{\partial\gamma}{\partial t}=\frac{\partial V_y/\partial t}{V} \quad (9)$$

$$\frac{\partial\gamma}{\partial t}=\sin^{-1}\left(\frac{a_y}{V}\right)=\sin^{-1}\left(\frac{L\cos\gamma-W+T\sin\gamma-D\sin\gamma}{Vm}\right) \quad (10)$$

$$\frac{\partial h}{\partial t}\frac{\partial t}{\partial x}=\frac{\partial h}{\partial x}=\tan\gamma \quad (11)$$

Our initial values are the velocity and the path angle so an expression, which combines both initial values, is needed.

$$\frac{\partial\gamma}{\partial t}\frac{\partial t}{\partial V}=\frac{\sin^{-1}\left(\frac{a_y}{V}\right)}{\sqrt{a_x^2+a_y^2}} \quad (12)$$

The trajectory is found by:

- The integration of *Eq.(12)* along the path gives the path angle γ for each velocity
- Using the inverse of *Eq.(8)* and integrating it along the velocity will give the horizontal distance along x
- Finally using and integrating *Eq.(11)* we have the vertical distance for the corresponding horizontal distance.

III. STAGE 2: THE ASCENT

The same method is repeated for the ascent part. The only difference is that Eqs.(1,2) are replaced by (13-14).

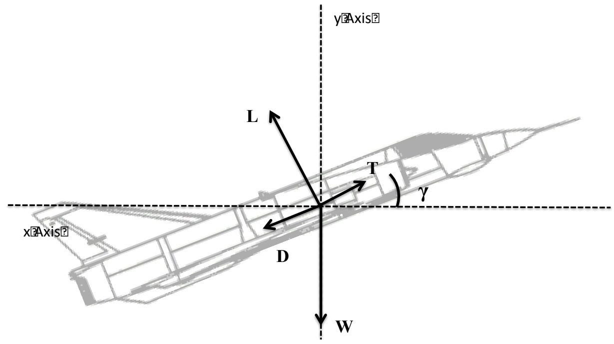

Figure 2. Fig. 1 Body diagram second phase

$$\sum F_x = T\cos\gamma - D\cos\gamma - L\sin\gamma = ma_x \quad (13)$$

$$\sum F_y = L\cos\gamma - W + T\sin\gamma - D\sin\gamma = ma_y \quad (14)$$

IV. COMPUTATION OF TTOTAL, DTOTAL, AND MAV OF THE PERIODIC CYCLE:

Descent: $$t_d=\int\frac{\partial x_d}{V\cos\gamma} \quad (15)$$

Ascent time: $$t_a=\int\frac{\partial x_a}{V\cos\gamma} \quad (16)$$

Cycle time: $$t_{total}=t_d+t_a \quad (17)$$

Descent distance: $$d_d=\int\sqrt{dx_d^{\ 2}+dh_d^{\ 2}} \quad (18)$$

Ascent distance: $$d_a=\int\sqrt{dx_a^{\ 2}+dh_a^{\ 2}} \quad (19)$$

cycle distance: $$d_{total}=d_d+d_a \quad (20)$$

Where the expression $\sqrt{dx_d^2+dx_a^2}$ is a small displacement in the periodic trajectory.

Average Velocity: $$V_{av}=\frac{d_{total}}{t_{total}} \quad (21)$$

Average Mach No. $$M=\frac{V_{av}}{s} \quad (22)$$

V. THE ENERGY BALANCE

We are looking for the amount of energy the aircraft will use in one full cycle. As the thrust is the only force requiring energy from the aircraft only the work created by the thrust W_T and the lost potential Energy E_h are taken into account.

The work due to the thrust is for the descent (first stage):

$$dW_{Td}=T(M_0)\sqrt{dx^2+dh^2} \quad (23)$$

For the ascent (second stage):

$$dW_{Ta}=T(M_0)F_T\sqrt{dx^2+dh^2} \quad (24)$$

Where $\sqrt{dx^2+dh^2}$ is a small displacement in the periodic trajectory. The integration of *Eq.(23)* and *Eq.(24)* gives the total work done by the thrust over the whole trajectory.

We take the thrust to be constant with a value of $T(M_0)$ for the descent and has a value of $T(M_0)\times F_T$ for the ascent. The initial conditions at the beginning of the descent obviously cannot be retrieved at the end of the climb without adding energy. This extra energy consumption is equal to the potential energy lost to drag and is represented by the thrust factor F_T, which therefore is always bigger than one in order to retrieve the initial conditions of both altitude and Mach number at the end of the cycle.

The total energy used by the aircraft in the periodic maneuver is

$$E_{T_p} = W_{Tp} = W_{Tpd} + W_{Tpa} \tag{25}$$

VI. COMPARISON WITH STEADY CRUISE

In order to prove a possible advantage of the periodic cycle over steady cruise we compare the cost of these two types of motion.

The equations of motion for steady cruise are, under the following assumptions:

- Constant altitude, density and temperature.
- Constant thrust, drag, lift and weight.
- Constant velocity equal to the average velocity of the periodic cycle

$$\sum F_x = T - D = 0 \Rightarrow T = D \tag{27}$$

$$\sum F_y = L - W = 0 \Rightarrow L = W \tag{28}$$

As there is no altitude change, the total energy consumed by the aircraft is equal to the work done due to the thrust

$$E_{Ts} = W_{Ts} = T(M_{av})x_{total} \tag{29}$$

Where x_{total} is the horizontal distance of the periodic trajectory.

We now calculate the ratio of the energies required R.

$$R = \frac{E_{T_S}}{E_{T_P}} \tag{30}$$

If this ratio R is larger than one, it means that the periodic cruise is better energetically than steady cruise.

WIND TUNNEL TEST

A parametric study was performed, and we show a typical result here. The periodic cruise chosen starts with a Mach number of *0.7* and acceleration during descent, until a maximum Mach number *1.1* is obtained. The plane then rotates and starts climbing back to the original altitude while decelerating. The numerical simulation computes all the variables of the trajectory, the energy spent by the aircraft and compares with a steady cruise at the average velocity for the same horizontal distance. As our nominal aircraft, we use an AGARD B [6] model (Fig 3), in order to be able to compare to wind tunnel results.

The lift, drag and pitching moment characteristics of the AGARD calibration model "B" were obtained in the Technion's *60 cm* x *80 cm* transonic wind tunnel, at Mach numbers from *0.2* to *1.14*. The measured data compared well with values obtained in other wind tunnels in all speed ranges (subsonic, transonic and supersonic). Ref 6 serves as the database of aerodynamics coefficients of the chosen model in order to create the periodic simulations as well as to compare results with steady flight.

The assumptions for this computational trajectory simulation are:

- The computation goes from *Mach 0.7 to Mach 1.1*.
- For comparison with the wind tunnel data the reference altitude is sea level, so the corresponding air density is $\rho(h) = 1.223\ kg/m^3$ and corresponding speed of sound is $s = 340\ m/s$
- For the same reason, this simulation does not take into account the change of air density due to the altitude lost. This will be shown a *posteriori* to be relatively small.

This simulation assumed constant lift and constant thrust. For the periodic case we assume that *Thrust Descent* $= T(M_0) = D(M_0)$, *Thrust Ascent* $= T(M_0) \times F_T$ and for steady state *Thrust*$=T(M_{av})$.

The AGARD model "B" has been a standard for calibration in the transonic and supersonic region for many years. It is a delta wing & body configuration. The geometry of the AGARD Model "B" is proportional to its body diameter only, which makes easy to configure it at any scale for any kind of wind tunnel size. The model we used has a body diameter $D = 0.046m$ and a mass $m = 1.5535kg$.

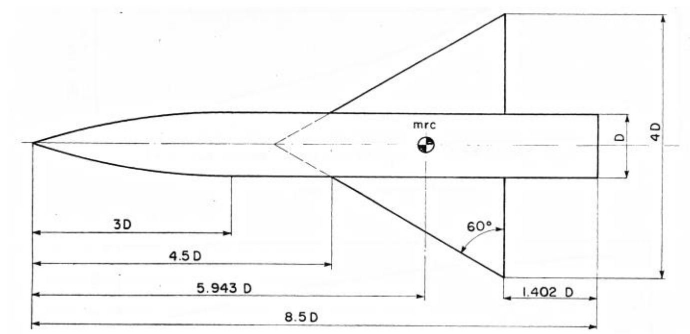

Figure 3. AGARD Model B Schematic (From ref. 6)

Like the steady path, periodic trajectory needs to be designed with a specific configuration of the lift, drag and angle of attack. One of the major constraints is to have constant vertical lift during the whole periodic cycle. The coefficient of lift and therefore the lift varies with Mach number and trajectory angle. Thus, to keep the vertical force constant while descending and accelerating, we change the angle of attack.

We therefore found α versus the Mach number for fixed lift equal to the AGARD model "B" weight. Then the drag force as a function of the Mach number for a same fixed lift is computed, using the following procedure:

- Take the lift versus Mach number for different α *(from* $\alpha=0°$ *to* $\alpha=4°$*)*
- Take a fixed lift value $(L = W = mg = 15.2\ N)$

- Make an interpolation between *α, L* and the *M*.
- Make an interpolation between each drag value and the corresponding *α* and *M*

Fig.4 shows the lift as a function of the Mach number for different angles of attack.

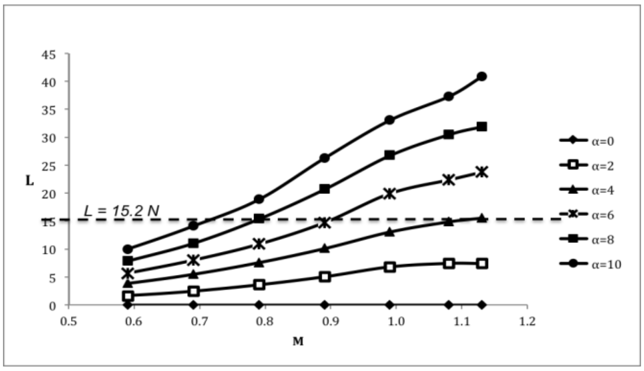

Figure 4. AGARD Model B lift as function of M for different AOA. Each dot is an experimental point from Khen et al.

The dashed-line shows the fixed lift (*L = W = mg = 15.2 N*), we can observe that this line crosses all lift curves from *L(4°)* to *L(0.5°)* and therefore in order to keep the lift constant we need to decrease the angle of attack when M is increasing. By taking the crossing points between the dashed-line and the lift curve, it is possible to create a function of *α(M)* for fixed lift.

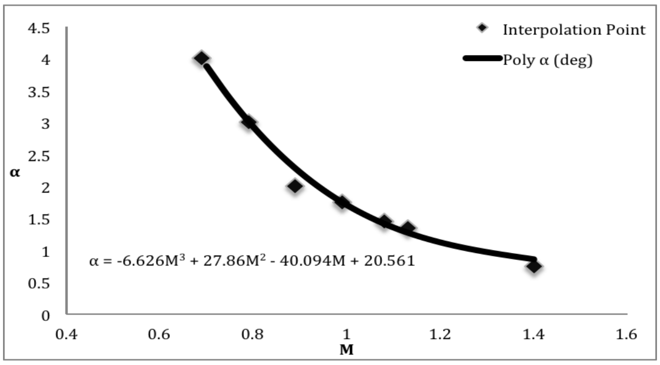

Figure 5. α as function of M for fixed L

A polynomial basic fitting curve has been added to *Fig.5* in order to get an equation for the variation of α with respect to the Mach number for a fixed lift. We observe a high rate of change between *M=0.7* and *M=1.1*.

Using *Fig.4* and *Fig.5* the computation of the drag coefficient for fixed lift can be done, the following *Fig.6* shows the functions $C_D(M)$ for fixed lift.

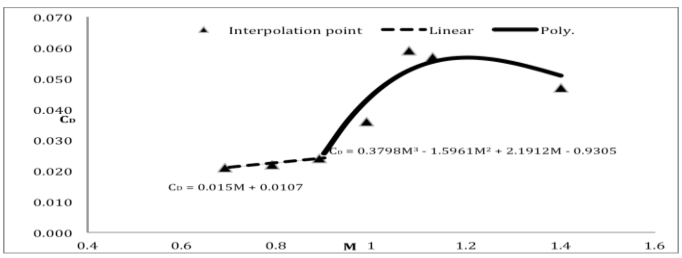

Figure 6. C_D as function of M for a fixed L

The drag coefficient curve can be divided into two parts. The first one is linear from *M=0.7* to *M=0.9* and the second one polynomial from *M=0.9* to *M=1.4*. This dissection allows us to be more precise with the calculations linked to this curve.

VII. RESULTS

Using the assumptions and the equations listed above the numerical simulation gave the following results: a total cycle time of *7.7 sec*, Mach average of *0.8*, a horizontal and vertical distance ration of *31/1.2km*, and a energy ratio *R* of *1.18 (18)*

Of course these results represent a maximum and idealistic energy gain. While this simulation does not take into account several loss factors, on the other hand, it is a simple linear trajectory, which probably can be improved on too. In any case, it shows that saving energy by doing periodic maneuvers is possible.

Moreover the average Mach number is quite low, this is due to the fact that the second phase (ascent) is much longer than the first phase (descent) and so the total cycle time reduces the average Mach number. Additionally the periodic trajectory is relatively flat due to the ratio of the Δh over the x_{total}.

Fig.7 confirms the fact that the second phase (ascent) is much longer than the first phase (descent).

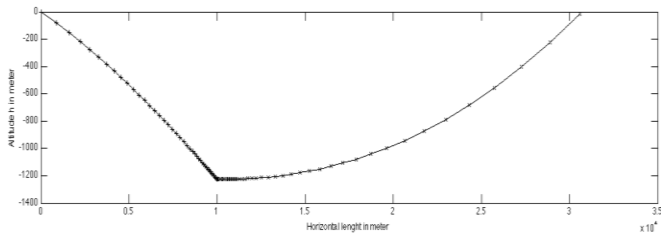

Figure 7. Full Periodic Trajectory. Separation between dots are the increments in time

These simulations show that a periodic cruise has real possibility of energy gain over steady state cruise. Since these simulations were made with many simplifying assumptions, we verify the above results by wind-tunnel test. We made a series of runs that simulate the acceleration and change of the angle of attack of a total cycle The Mach number increases until it reaches *M= 1.1* and then it decreases in a similar way until it gets back to *M= 0.7*. It's interesting to see in *Fig.8* that the drag force is always higher in the deceleration phase (ascent) phase than in the acceleration (descent) phase. This difference increases with the Mach number. The result is probably due to the buildup of a shock wave on the wing, during acceleration, so that they pre-exist when decelerating.

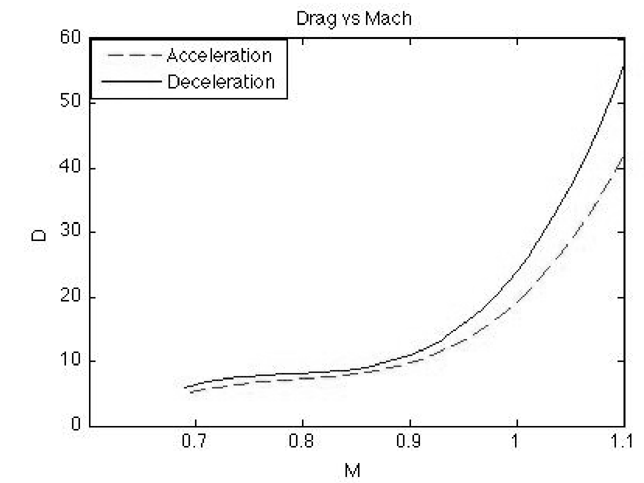

Figure 8. Wind tunnel drag of the AGARD -B model as function of M

Next, we compare the actual result of each case (simulation and wind tunnel). The following *Table I* makes a summary of the results of the simulation and of the wind tunnel test.

TABLE I. EXPERIMENTAL AND NUMERICAL RESULTS

	Mach	M_{av}	x_{tot} *km*	*Δh* *km*	*t* *sec*	F_T	*R*
Numerical	0.7-1.1	0.80	32	1.29	7.7	1.23	1.18
Experimental	0.7-1.1	0.80	32	1.29	7.7	1.23	1.26

Table I shows clearly similar results between the numerical simulation and the wind-tunnel experiment. Surprisingly, the experimental results are "better" than the numerical simulation. There is a *26%* energy gain using the wind tunnel trajectory and *18%* gain using the simulation trajectory. We hypothesize that the difference between the simulations and the experiment is, at least partially due to the drag hysteresis due to the finite time for shock formation [8] which was not included in the simulation.

It is important to reiterate that this result is idealistic and will probably not be obtained in real flight. For example, we need to consider the energy used in the pull up turns at the transitions, between the descent stage and the ascent. An estimate of this energy cost can be made as follows:

The airplane has the last path angle of the first phase then suddenly needs to be at the initial angle of the second phase (which is zero degree). For this pull-up manoeuvre, the flight path becomes curved in the vertical plane, with a turn rate ω.

The radius of curvature is given by:

$$r_{curve} = \frac{V^2}{g(n - 1)} \tag{33}$$

So the turn rate ω is given by:

$$w = \frac{dg}{dt} = \frac{V}{r_{curve}} = \frac{g(n - 1)}{V} \tag{34}$$

Then the arc

$$A_{arc} = r_{curve}\Delta\gamma \tag{35}$$

So the work done by the thrust during this transition is given by the following equation

$$W_{curve} = T \cdot A_{arc} \tag{36}$$

$$E_{Tp} = W_{Tp} = W_{Tpd} + W_{Tpa} + W_{curve} \tag{37}$$

In order to see the impact of this correction on the energy ratio R of the wind tunnel model and of the simulation it is important to notice that this pull up depends on the load factor n.

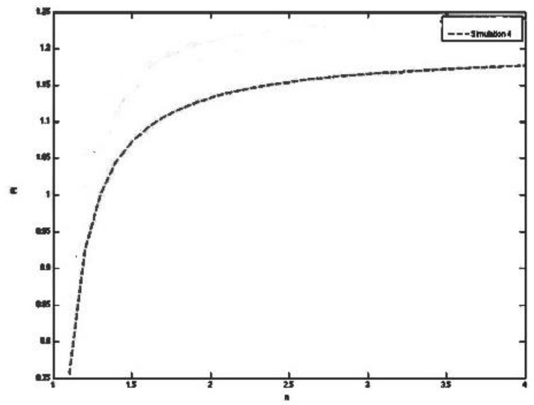

Figure 9. Energy ratio R as function of the load factor n

The load factor range from *1* to *4* has been chosen accordingly with typical load factors for military aircraft.

It is obvious from *Fig. 9* that this turn is affecting the energy ratio. We can see that when the load factor is close to one the ratio *R* is below one and so the periodic path is not saving energy anymore. However for values of over 2.5, the theoretical savings are approached.

VIII. CONCLUDING REMARKS

Periodic cruise with altitude variation shows a real possibility of advantage over steady level cruise with a possible energy gain of *16%* in the example case, retrieved both in wind tunnel experiments and corresponding analytic calculations. Despite all the assumptions that make the results less realizable, the high percentage of gain *(26%)* is high enough to promise that even if some assumptions will reduce the actual results, this number will stay positive.

This result is robust enough to encourage further study, of more complex cyclic motions, and full aircraft configurations, to identify optimal strategies and loss factors for specific aircraft.

NOMENCLATURE

a	=	Acceleration [LT^{-2}]
CL	=	Lift Coefficient
CD	=	Drag Coefficient
D	=	Drag Force [MLT^{-2}]

E = Total Energy [ML2 T^{-2}]

FT = Thrust Factor

g = Gravity Acceleration [LT^{-2}]

h = Vertical Distance [L]

L = Lift Force [MLT^{-2}]

m = AGARD Model B Mass [M]

M = Mach Number

n = Load factor

r = Radius [L]

R = Energy Ratio

s = Speed of Sound [L/T]

t = Time [T]

V = Velocity [L/T]

W = Weight [MLT^{-2}]

WT = Thrust Work [ML^2 T^{-2}]

x = Horizontal Distance Altitude [L]

α = Angle of Attack (AOA)[deg.]

γ = Path Angle [deg.]

ρ_∞ = Air Density [M/L^3]

ω = Turn rate [1/T]

Subscripts

a = Ascent

arc = Arc

av = Average

curve = pull- up curve

d = Descent

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